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AN ALGORITHMIC APPROACH FOR SHORTEST PATH PROBLEM BY POSSIBILITY MEASURE WITH TYPE-2 FUZZY NUMBER

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ABSTRACT

Type-2 fuzzy sets are a generalization of the ordinary sets in which each type-2 fuzzy set is characterized by a fuzzy membership function. In this paper we proposed an algorithm for finding shortest path and shortest path length using possibility measure based on extension principle. In a network each edge have been assigned as discrete type-2 fuzzy number.

KEYWORDS: Type-2 Fuzzy Number Possibility Measure Discrete Type-2 Fuzzy Number Extension Principle

1. INTRODUCTION

The shortest path problem concentrates on finding the path with minimum distance to find the shortest path from source node to destination node is a fundamental matter in graph theory.

The fuzzy shortest path problem was first analyzed by Dubois and Prade [4]. Okada and Soper [9] developed an algorithm based on the multiple labeling approach, by which a number of non dominated paths can be generated. Type-2 fuzzy set was introduced by Zadeh [13] as an extension of the concept of an ordinary fuzzy set. The type-2 fuzzy logic has gained much attention recently due to its ability to handle uncertainty, and many advances appeared in both theory and applications.

In general in a directed acyclic network, crisp values are widely used as the weights on edges, but there are many cases when we cannot determine these weights precisely. In these cases, we can use fuzzy weights instead of crisp weights to express the uncertainty, and type-2 fuzzy weights will be more suitable if this uncertainty varies under some conditions.

The concept of fuzzy measures was introduced by sugeno [11], Good surveys of various types of measures subsumed under this broad concept were prepared by Dubois and Prade [3], Bacon[2], and Wierzchon [12]. There were many researches using different approaches, but we will mainly focus on an approach based on possibility theory proposed by Okada[10].

The structure of paper is following: In Section 2, we have some basic concepts required for analysis. Section 3, gives an algorithm to find shortest path and shortest path length with type-2 fuzzy number using possibility measure. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5. Finally in section 6, conclusion i included.

2. CONCEPTS

2.1 Type-2 Fuzzy Set

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where x ∈ X and u ∈ $J_x \subseteq [0,1]$.

ie, $\tilde{A}$ = {((x,u), $\mu_{\tilde{A}}(x,u)$) / ∀ x ∈ X, ∀ u ∈ $J_x \subseteq [0,1]$ } in which 0 ≤ $\mu_{\tilde{A}}(x,u)$ ≤ 1. $\tilde{A}$ can be expressed as $\tilde{A}$ = $\int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2. Type-2 Fuzzy Number

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,

2. $\tilde{A}$ is a convex set,

3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.3. Discrete Type-2 Fuzzy Number:

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x\in X} \mu_{\tilde{A}}(\mathrm{x}) / \mathrm{x}$ where $\mu_{\tilde{A}}(\mathrm{x}) = \sum_{u\in J_x} f_x(\mathrm{u}) / \mathrm{u}$ where J_x is the primary membership.

2.4. Extension Principle

Let A_1, A_2, A_r be type-1 fuzzy sets in X_1, X_2, X_r, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets A_1, A_2, A_r a type-1 fuzzy set B on Y, through f, i. e, B = f(A_1, ...,A_r), such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1, x_2, \ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1), \ldots, \mu_{A_n}(x_n) \; iff^{-1}(y) \neq \phi \\ 0, \qquad f^{-1}(y) = \phi \end{cases}$$

2.5. Addition on Type-2 Fuzzy Numbers:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A} = \sum \mu_{\tilde{A}}(x) / \mathrm{x}$ and $\tilde{B} = \sum \mu_{\tilde{B}}(\mathrm{y}) / \mathrm{y}$ where $\mu_{\tilde{A}}(x) = \sum f_x(\mathrm{u}) / \mathrm{u}$ and $\mu_{\tilde{B}}(x) = \sum g_y(\mathrm{w}) / \mathrm{w}$. The addition of these two types-2 fuzzy numbers $\tilde{A} \oplus \tilde{B}$ is defined as

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z}) = \bigcup_{z=x+y} (\mu_{\tilde{A}}(\mathrm{x}) \bigcap \mu_{\tilde{B}}(\mathrm{y}))$$

$$= \bigcup_{z=x+y} ((\sum_i \mathrm{f}_x(\mathrm{u}_i)/\mathrm{u}_i) \bigcap (\sum_j g_y(\mathrm{w}_j)/\mathrm{w}_j))$$

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z}) == \bigcup_{z=x+y} ((\sum_{i,j} (\mathrm{f}_x(\mathrm{u}_i) \wedge g_y(\mathrm{w}_j)) / (\mathrm{u}_i \wedge \mathrm{w}_j))$$

2.6. Relative Posssibility

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then the relative possibility for av($\tilde{A}$) = x and av($\tilde{B}$)= y can be defined as

$$\mathrm{s}(av(\tilde{\mathrm{A}}) = x \wedge av(\tilde{\mathrm{B}}) = \mathrm{y}) = \sum\sum u.f(u).w.g(w)$$

Where $av(\tilde{\mathrm{A}})$ is the actual value of fuzzy set $\tilde{A}$ and $\tilde{A} = \sum f_x(\mathrm{u})/\mathrm{u}/\mathrm{x}$ and $\tilde{B} = \sum g_y(\mathrm{w})/\mathrm{w}/\mathrm{y}$.

2.7. Possibility Measure

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then the possibility that $\tilde{A} < \tilde{B}$ is defined as

$$s(\tilde{A} < \tilde{B}) = \frac{\sum_{(x,y)\in\Omega(\tilde{A}<\tilde{B})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}{\sum_{(x,y)\in\Omega(\mathrm{T})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}$$

Where

$\Omega(\tilde{A} < \tilde{B})$ is the set of all possible situations where $\mathrm{av}(\tilde{A}) < av(\tilde{B})$.

$\Omega(\mathrm{T}) = \Omega(\tilde{\mathrm{A}} < \tilde{\mathrm{B}}) + \Omega(\tilde{\mathrm{A}} = \tilde{\mathrm{B}}) + \Omega(\tilde{\mathrm{A}} > \tilde{\mathrm{B}})$.

Similarly

$$s(\tilde{A} > \tilde{B}) = \frac{\sum_{(x,y)\in\Omega(\tilde{A}>\tilde{B})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}{\sum_{(x,y)\in\Omega(\mathrm{T})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)} \text{ and}$$

$$s(\tilde{A} = \tilde{B}) = \frac{\sum_{(x,y)\in\Omega(\tilde{A}=\tilde{B})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}{\sum_{(x,y)\in\Omega(\mathrm{T})} s(av(\tilde{A}) = x \wedge av(\tilde{B}) = y)}.$$

2.8. Degree of Possibility

The Degree of possibility D_p for the path P is defined as

$$D_p = \min_{p^1 \in p}\{s(p_i \le p_j) + s(p_i = p_j)\}.$$

3. ALGORITHM

Step 1: Computation of Possible Paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1, 2, n for possible n paths.

Step 2: Computation of Relative Possibilities

$\tilde{A} = \sum \mu_{\tilde{A}}(x)/\text{x}$ and

$\tilde{B} = \sum \mu_{\tilde{B}}(\text{y})/\text{y}$

Compute relative possibility $s\ (av(\tilde{p}_i(x_k)) \wedge av(\tilde{p}_j(\text{x}_k)))$ for all situations where x_k is the actual value of the path $\tilde{p}_i$ and j = i + 1.

Step 3: Computation of Possibility Measure

i. i = 1

ii. j = i+1

iii. Compute $s_{ij}(\tilde{p}_i < \tilde{p}_j)$ using definition 2.7

iv. Write $s_{ij}(\tilde{p}_i < \tilde{p}_j)$

v. Put j = j +1

vi. If j≤ n then go to (iii)

vii. Put i = i + 1

viii. If i<n then go to (ii)

ix. Compute the possibility of $s_{ij}(\tilde{p}_i = \tilde{p}_j)$

Step 4: Computation of Degree of Possibility

$D(\tilde{p}_i) = \min\{s_{ij}(\tilde{p}_i < \tilde{p}_j) + (1/2)s_{ij}(\tilde{p}_i = \tilde{p}_j)\}$ where i = 1,2, n and j = i+1.

Step 5: Shortest Path and Shortest Path Length

The path which is having highest degree of possibility is the Shortest path and the corresponding path length is the Shortest path Length.

4. NETWORK TERMINOLOGY

Consider a directed network G (V, E) consisting of a finite set of nodes V = {1,2, n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i, j), where i, j ∈ V and $i \neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence $P_{ij} = \{i = i_1, (i_1, i_2), i_2, i_{l-1}, (i_{l-1}, i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{si} in G (V,E) is assumed for every node $i \in V - \{s\}$.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i, j \in P)} \tilde{d}_{ij}$

5. NUMERICAL EXAMPLE

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 7 edges with type-2 fuzzy number.

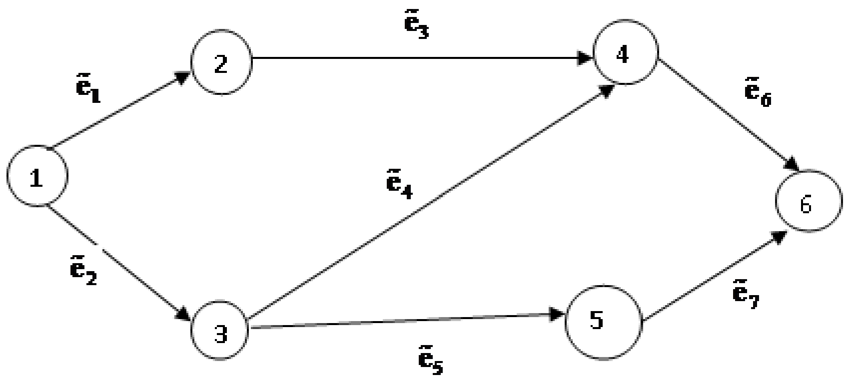

Figure 5.1

Solution:

The edge Lengths are

$\tilde{e}_1$ = (0.5/0.2+0.4/0.3)/2 + (0.4/0.2)/3

$\tilde{e}_2$ = (0.3/0.2 + 0.8/0.3)/1 + (0.2/0.8)/3

$\tilde{e}_3$ = (0.7/0.2)/2 + (0.9/0.4 +0.7/0.5)/4

$\tilde{e}_4$ = (0.6/0.2)/4

$\tilde{e}_5$ = (0.9/0.4 +0.7/0.5)/3 + (0.4/0.7)/5

$\tilde{e}_6$ = (0.8/0.3 + 0.4/0.5)/2

$\tilde{e}_7$ = (0.6/0.4)/2 + (0.7/0.5+ 0.5/0.6)/4

Step 1: Computation of Possible Paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1, 2, n for possible n paths.

$\tilde{p}_1 = \tilde{e}_1 + \tilde{e}_3 + \tilde{e}_6 = 1 - 2 - 4 - 6$

$\tilde{p}_2 = \tilde{e}_2 + \tilde{e}_4 + \tilde{e}_6 = 1 - 3 - 4 - 6$

$\tilde{p}_3 = \tilde{e}_2 + \tilde{e}_5 + \tilde{e}_7 = 1 - 3 - 5 - 6$

$\tilde{L}_1$ = (0.5/0.2)/6 + (0.4/0.2)/7 + (0.5 /0.2 +0.4/0.3)/8 + (0.4/0.2)/9

$\tilde{L}_2$ = (0.6/0.2)/7 + (0.2/0.2)/9

$\tilde{L}_3$ = (0.3/0.2 + 0.6/0.3)/6 + (0. 2/0.4)/8 + (0.2/0.4+0.2/0.5)/10 + (0.2/0.5 + 0.2/0.6)/12

Step 2: Computation of Relative Possibilities

$\tilde{A} = \sum \mu_{\tilde{A}}(x) / \mathrm{x}$ and

$\tilde{B} = \sum \mu_{\tilde{B}}(\mathrm{y}) / \mathrm{y}$

Compute relative possibility $s\ (av(\tilde{p}_i(x_k)) \wedge av(\tilde{p}_j(\mathrm{x}_k)))$ for all situations where x_k is the actual value of the path $\tilde{p}_i$ and j = i + 1.

$s(\tilde{p}_1 = 6 \wedge \tilde{p}_2 = 7)$ = 0.012 $s(\tilde{p}_1 = 6 \wedge \tilde{p}_2 = 9)$ = 0.004

$s(\tilde{p}_1 = 6 \wedge \tilde{p}_3 = 6)$ = 0.024 $s(\tilde{p}_1 = 6 \wedge \tilde{p}_3 = 8)$ = 0.008

$s(\tilde{p}_1 = 6 \wedge \tilde{p}_3 = 10)$ = 0.018 $s(\tilde{p}_1 = 6 \wedge \tilde{p}_3 = 12)$ = 0.022

$s(\tilde{p}_1 = 7 \wedge \tilde{p}_2 = 7)$ = 0.0096 $s(\tilde{p}_1 = 7 \wedge \tilde{p}_2 = 9)$ = 0.0032

$s(\tilde{p}_1 = 7 \wedge \tilde{p}_3 = 6)$ = 0.0192 $s(\tilde{p}_1 = 7 \wedge \tilde{p}_3 = 8)$ = 0.0064

$s(\tilde{p}_1 = 7 \wedge \tilde{p}_3 = 10)$ = 0.014 $s(\tilde{p}_1 = 7 \wedge \tilde{p}_3 = 12)$ = 0.0176

$s(\tilde{p}_1 = 8 \wedge \tilde{p}_2 = 7)$ = 0.0264 $s(\tilde{p}_1 = 8 \wedge \tilde{p}_2 = 9)$ = 0.0088

$s(\tilde{p}_1 = 8 \wedge \tilde{p}_3 = 6)$ = 0.0528 $s(\tilde{p}_1 = 8 \wedge \tilde{p}_3 = 8)$ = 0.0176

$s(\tilde{p}_1 = 8 \wedge \tilde{p}_3 = 10)$ = 0.0396 $s(\tilde{p}_1 = 8 \wedge \tilde{p}_3 = 12)$ = 0.0484

$s(\tilde{p}_1 = 9 \wedge \tilde{p}_2 = 7)$ = 0.0096 $s(\tilde{p}_1 = 9 \wedge \tilde{p}_2 = 9)$ = 0.0032

$s(\tilde{p}_1 = 9 \wedge \tilde{p}_3 = 6)$ = 0.0192 $s(\tilde{p}_1 = 9 \wedge \tilde{p}_3 = 8)$ = 0.0064

$s(\tilde{p}_1 = 9 \wedge \tilde{p}_3 = 10)$ = 0.0224 $s(\tilde{p}_1 = 9 \wedge \tilde{p}_3 = 12)$ = 0.0176

$s(\tilde{p}_2 = 7 \wedge \tilde{p}_3 = 6)$ = 0.0288 $s(\tilde{p}_2 = 7 \wedge \tilde{p}_3 = 8)$ = 0.0096

$s(\tilde{p}_2 = 7 \wedge \tilde{p}_3 = 10)$ = 0.0216 $s(\tilde{p}_2 = 7 \wedge \tilde{p}_3 = 12)$ = 0.0264

$s(\tilde{p}_2 = 9 \wedge \tilde{p}_3 = 6)$ = 0.0096 $s(\tilde{p}_2 = 9 \wedge \tilde{p}_3 = 8)$ = 0.0032

$s(\tilde{p}_2 = 9 \wedge \tilde{p}_3 = 10)$ = 0.0072 $s(\tilde{p}_2 = 9 \wedge \tilde{p}_3 = 12)$ = 0.0088

Step 3: Computation of Possibility Measure

- i = 1
- j = i+1
- Compute $s_{ij}(\tilde{p}_i < \tilde{p}_j)$ using definition 2.7
- Write $s_{ij}(\tilde{p}_i < \tilde{p}_j)$
- Put j = j +1
- If j≤ n then go to (iii)
- Put i = i + 1
- If i<n then go to (ii)
- Compute the possibility of $s_{ij}(\tilde{p}_i = \tilde{p}_j)$

$s_{12}(\tilde{p}_1 < \tilde{p}_2)$ = 0.0513 $s_{13}(\tilde{p}_1 < \tilde{p}_3)$ = 0.393

$s_{12}(\tilde{p}_1 = \tilde{p}_2)$ = 0.0235 $s_{13}(\tilde{p}_1 = \tilde{p}_3)$ = 0.076

$s_{21}(\tilde{p}_2 < \tilde{p}_1)$ = 0.066 $s_{21}(\tilde{p}_2 = \tilde{p}_1)$ = 0.0235

$s_{23}(\tilde{p}_2 < \tilde{p}_3)$ = 0.1349 $s_{23}(\tilde{p}_2 = \tilde{p}_3)$ = 0

Step 4: Computation of Degree of Possibility

$D(\tilde{p}_i) = \min\{s_{ij}(\tilde{p}_i < \tilde{p}_j) + (1/2)s_{ij}(\tilde{p}_i = \tilde{p}_j)\}$ where i = 1,2, . . .n and j = i+1.

$D(\tilde{p}_1)$ = min {0.0631, 0.431} = 0.0631

$D(\tilde{p}_2)$ = min {0.0778, 0.1349} = 0.0778

$D(\tilde{p}_3)$ = min {0.076, 0.2551} = 0.076

Step 5: Shortest Path and Shortest Path Length

The path which is having highest degree of possibility is the Shortest path and the corresponding path length is the Shortest path Length.

Since Highest degree is $\tilde{p}_2$ ie. 0.0778.

We conclude that the Shortest Path is 1 – 3 – 4 – 6 and the shortest length is

$\tilde{L}_2$ = (0.6/0.2)/7 + (0.2/0.2)/9.

6. CONCLUSIONS

In this paper we have developed an algorithm for finding shortest path and shortest path length using possibility measure with type-2 fuzzy number. This is the different approach for finding shortest path in a network, with possibility measure.

REFERENCES

1. V. Anusuya and R, Sathya, "Type-2 fuzzy shortest path", International Journal of fuzzy mathematical Archive, Vol. 2, 2013, 36-42.
2. Bacon, G., "Distinction between several subsets of fuzzy measures", Fuzzy sets and systems, 5, 1981, pp. 291 – 305.
3. D. Dubois and H. Prade, "Fuzzy sets and systems; Theory and applications", Academic press, New York.
4. D. Dubois and H. Prade, Theory and applications : Fuzzy Sets and Systems, Man and Cybernetics, Vol. 23, No. 2, 1993, pp. 403 – 405
5. George J. Klir and Tina A. Folger, " Fuzzy sets, Uncertainty and Information", Prentice Hall of India Private Limited, New Delhi, 1995
6. Jan Caha, Jiri Dvorsky, "Optimal Path Problem with possibilistic weights", GIS Ostrava, Jan 2014, 27-29.
7. S. Lee and K.H. Lee, "Shortest Path Problem in a Type – 2 Weighted Graph", J. of korea Fuzzy and Intelligent Systems Society, Vol.11, No.6, pp. 528 – 531, 2001.
8. Nagoorgani, A. and Anusuya, V. Fuzzy Shortest Path by Linear Multiple Objective Programming, Proceedings of the International Conference On Mathematics and Computer Science, , Loyola College, Chennai, Jan 10 and 11, 2005, pp.287 – 294.
9. S. Okada and T. Soper, "A Shortest Path Problem On a Network with Fuzzy Arc Lengths", Fuzzy Sets and Systems, 109, 2000, 129 – 140.
10. Shinkoh Okada, "Interactions among paths in Fuzzy shortest path problems", Proc.of the Joint 9th IFSA world congress and 20th NAFIPS International Conference, 2001, pp.41-46.

11. Sugeno,M. "Fuzzy measures and Fuzzy integrals- a survey", In Gupta, saridis, and Gaines, 1977, pp.89-102.

12. Wierzchon, S. T., "On fuzzy measure and fuzzy integral", In: gupta and Sanchez, 1982, pp.79-86.

13. L.A. Zadeh, "The Concept of a Linguistic Variable and its Application to Approximate Reasoning – 1", Inform. Sci. 8 (1975) 199 – 249.